Predicting Fatigue Life of Welded Joints under Multiaxial Loads

Eric Krupitzer
DaimlerChrysler Corporation
Acknowledgments

• Dr. Yung-Li Lee
  – Senior Specialist
• Tana Tjhung
  – Product Development Chassis Engineer
• Algernon Jordan
  – Product Development Body Engineer
Outline

- Proportional Loading
- Nonproportional Loading
- Nonproportional Hardening
- Critical Plane vs. Equivalent Stress
- LTJ (Lee, Tjhung, Jordan) Fatigue Damage Model
- Validation Studies
- Conclusions
- Questions
• What does it mean?
  – Fixed local principal stress/strain axes (e.g., in-phase loading)

Local Axial Strain

Local Shear Strain
Nonproportional Loading

- What does it mean?
  - Local principal stress/strain axes rotate with time (e.g., out-of-phase loading)

Local Axial Strain

Local Shear Strain

\[ \epsilon_x \]

\[ \gamma_{xy} \]
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point B, \( \phi = 17^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point C, \(\phi = 32^\circ\)
**Example: Out-of-phase tension/torsion loading**

\[ \sigma_x = 100 \cos(2\pi t) \]

\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point D, \( \phi = 45^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \frac{100}{\sqrt{3}} \cos(2\pi t - \pi/2) \]
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point F, \( \phi = 73^\circ \)
Example: Out-of-phase tension/torsion loading

$$\sigma_x = 100 \cos(2\pi t)$$
$$\tau_{xy} = \frac{100}{\sqrt{3}} \cos(2\pi t - \pi/2)$$
Rotation of Principal Stress Axes

Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point H, \( \phi = -73^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]

\[ \tau_{xy} = \frac{100}{\sqrt{3}} \cos(2\pi t - \pi/2) \]

Point I, \( \phi = -58^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \frac{100}{\sqrt{3}} \cos(2\pi t - \pi/2) \]

Point J, \( \phi = -45^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]

Point K, \( \phi = -32^\circ \)
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2\pi t - \pi/2) \]
Example: Out-of-phase tension/torsion loading

\[ \sigma_x = 100 \cos(2\pi t) \]
\[ \tau_{xy} = \left(\frac{100}{\sqrt{3}}\right) \cos(2t - \pi/2) \]

Point M=A, \( \phi = 0 \)
Fatigue Strength Reduction

$\sigma_{eq,a}$ (or $\varepsilon_{eq,a}$)

$\sigma_{eq,a,P}$

$\sigma_{eq,a,NP}$

$N_f$

$N$

90° out-of-Phase (Nonproportional)

In-Phase (Proportional)
Fatigue Life Reduction

\[ \sigma_{eq,a} \ (\text{or } \varepsilon_{eq,a}) \]

\[ \sigma_{\text{applied}} \]

90° out-of-Phase
(Nonproportional)

In-Phase
(Proportional)

\[ N_{OP} \quad N_{IP} \]
Nonproportional Hardening

Proportional (In-phase)

Nonproportional (Out-of-Phase)
Nonproportional Hardening Effects

• Result of Nonproportional Hardening
  – Decreases
    • fatigue life
    • strength
  – Few models account for nonproportional hardening effects
Critical Plane vs. Equivalent Stress

• Critical Plane Approach
  – **Pro**: able to determine direction for crack initiation and propagation
  – **Con**: not accurate enough

• Equivalent Stress Approach
  – **Pro**: more accurate (accounts for Nonproportional Hardening using LTJ Model)
  – **Con**: not able to determine location for crack initiation and propagation
Critical Plane Approach
LTJ Fatigue Damage Model

- Expansion of Dr. Sonsino’s Model
- Developed by Dr. Yung-Li Lee, Tana Tjhung, and Algernon Jordan
- Accounts for nonproportional hardening effects
  - Sensitivity on shear-to-normal stresses
  - Mean offset
  - Material
  - Load-path history
\[ \sigma_{ef, NP}(N) = \sigma_{ef, P}(N) \cdot \left( \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right) \cdot (1 + \alpha \cdot f_{NP}) \]
\[
\sigma_{ef, NP}(N) = \sigma_{ef, P}(N) \cdot \left( \frac{\sigma_f'}{\sigma_f' - \sigma_m} \right) \cdot (1 + \alpha \cdot f_{NP})
\]

\[
\sigma_{ef, P}(N) = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \cdot \sigma_{y,a} + 3 \cdot \tau_{xy,a}^2} \times f_G^2
\]

“Size Effect Factor”

Shear-to- Normal Stress Sensitivity
Size Effect Factor, $f_G$

\[ f_G = 1.00 \]
Size Effect Factor, $f_G$

\[ f_G = 1.25 \]

Bending and Torsion
\[ \sigma_{ef, NP}(N) = \sigma_{ef, P}(N) \cdot \left( \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right) \cdot (1 + \alpha \cdot f_{NP}) \]
LTJ Fatigue Damage Model

Mean Offset, $\sigma_m$

Path A: Pure Bending
Path B: Pure Bending with mean offset
Path C: Pure Torsion
Path D: Pure Torsion with mean offset
Path F: Proportional Bending/Torsion
Path H: Torsion with constant Bending
Path I: Nonproportional Bending/Torsion
Path J: Nonproportional Bending/Torsion

$\sigma_m = 0$
LTJ Fatigue Damage Model

Mean Offset, $\sigma_m$

Path A: Pure Bending
Path B: Pure Bending with mean offset
Path C: Pure Torsion
Path D: Pure Torsion with mean offset
Path F: Proportional Bending/Torsion
Path H: Torsion with constant Bending
Path I: Nonproportional Bending/Torsion
Path J: Nonproportional Bending/Torsion

$\sigma_m > 0$
LTJ Fatigue Damage Model

- Mean Offset, $\sigma'_f$

\[ \sigma_{ef,P} \quad \sigma'_f \]

$y$-intercept with no mean offset

$N$
• Mean Offset

\[ \sigma_{ef,P} \]

\[ \sigma' \]

\[ \sigma'_f - \sigma_m \]

\[ \sigma_m \]

\[ N \]
• Mean Offset

\[
\sigma_{ef,P} \quad \sigma_f' \quad \sigma_f' - \sigma_m \quad N
\]
\[
\sigma_{ef, NP}(N) = \sigma_{ef, P}(N) \cdot \left( \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right) \cdot (1 + \alpha \cdot f_{NP})
\]
LTJ Fatigue Damage Model

Maximum shearing stress/strain planes

Interaction between slip systems

Loading
LTJ Fatigue Damage Model

\[ \alpha = \frac{\sigma_{NP}}{\sigma_P} - 1 \]

Effective Stress Amplitude, MPa

90° out-of-phase

Nonproportional hardening

In-phase

Effective Strain Amplitude
## LTJ Fatigue Damage Model

<table>
<thead>
<tr>
<th>Material</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>304 stainless steel</td>
<td>1.0</td>
</tr>
<tr>
<td>316 stainless steel</td>
<td>1.0</td>
</tr>
<tr>
<td>OFHC copper</td>
<td>1.0</td>
</tr>
<tr>
<td>316 stainless steel at 550 C</td>
<td>0.37</td>
</tr>
<tr>
<td>1045 steel</td>
<td>0.3</td>
</tr>
<tr>
<td>Inconel</td>
<td>0.2</td>
</tr>
<tr>
<td>42CrMo steel</td>
<td>0.15</td>
</tr>
<tr>
<td>1% CrMoV steel</td>
<td>0.14</td>
</tr>
<tr>
<td>En15R</td>
<td>0.14</td>
</tr>
<tr>
<td>1100 aluminum</td>
<td>0.0</td>
</tr>
<tr>
<td>6061 aluminum</td>
<td>0.05</td>
</tr>
<tr>
<td>7075 aluminum</td>
<td>0.0</td>
</tr>
</tbody>
</table>
\[ \sigma_{e_f, NP}(N) = \sigma_{e_f, P}(N) \cdot \left( \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right) \cdot (1 + \alpha \cdot f_{NP}) \]

Load-Path History
To make \( f_{NP} \) unity under 90 deg out-of-phase loading

\[
f_{NP} = \frac{C}{T \sigma_{1,\text{max}}} \int_{0}^{T} \left[ \sin \xi(t) |\sigma_1(t)\right] dt
\]

To normalize \( T \) and \( \varepsilon_{\text{max}} \) unity under 90 deg out-of-phase loading

Reference: Itoh, Ohnami, and Socie
## LTJ Fatigue Damage Model

<table>
<thead>
<tr>
<th>Case</th>
<th>( f_{NP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
</tr>
<tr>
<td>4</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>0.77</td>
</tr>
<tr>
<td>9</td>
<td>0.77</td>
</tr>
<tr>
<td>10</td>
<td>0.77</td>
</tr>
<tr>
<td>11</td>
<td>0.46</td>
</tr>
<tr>
<td>12</td>
<td>0.77</td>
</tr>
<tr>
<td>13</td>
<td>0.77</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
</tbody>
</table>

The diagrams represent different cases of fatigue damage with varying stress trajectories: Case 0, Case 1, Case 2, Case 3, Case 4, Case 5, Case 6, Case 7, Case 8, Case 9, Case 10, Case 11, Case 12, Case 13, and Case 14.
LTJ Fatigue Damage Model

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_{NP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
</tr>
<tr>
<td>4</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>0.77</td>
</tr>
<tr>
<td>9</td>
<td>0.77</td>
</tr>
<tr>
<td>10</td>
<td>0.77</td>
</tr>
<tr>
<td>11</td>
<td>0.46</td>
</tr>
<tr>
<td>12</td>
<td>0.77</td>
</tr>
<tr>
<td>13</td>
<td>0.77</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
</tbody>
</table>
Validation Studies

- Welded Joints under complicated loading
- First Validation Study (ASTM 519)
  - Validate load-path history, mean stress, and size effect factor
- Second Validation Study (StE 460)
  - Validate material and size effect factor
• First Validation Study

Tube-plate S-N curve for nonproportional and proportional loading paths, using the LTJ stress parameter
• Second Validation Study

Flange-Tube

Tube-Tube
• Second Validation Study

Normalized presentation of fatigue tests with different welded connections under uniaxial and multiaxial loading states

- Pure Bending
- Pure Torsion
- In-Phase Bending/Torsion
- Out-of-Phase Bending/Torsion

Material: StE 460
- \( R_{p0.2} = 520 \) MPa
- \( R_m = 670 \) MPa

Welding: MAG with unmachined and machined welds of flange-tube and tube-tube connections

State: Stress-relief annealed
Conclusions

• Nonproportional hardening results from nonproportional loading
  – Causes damaging effects which reduces fatigue life
  – Few models account for these damaging effects

• 3 tests required to use the LTJ model
  – Bending only
  – Torsion only
  – 90° out-of-phase

• Equivalent Stress Approach
  – Applies LTJ model to calculate damage in practical applications