

GREAT DESIGNS IN
STEEL

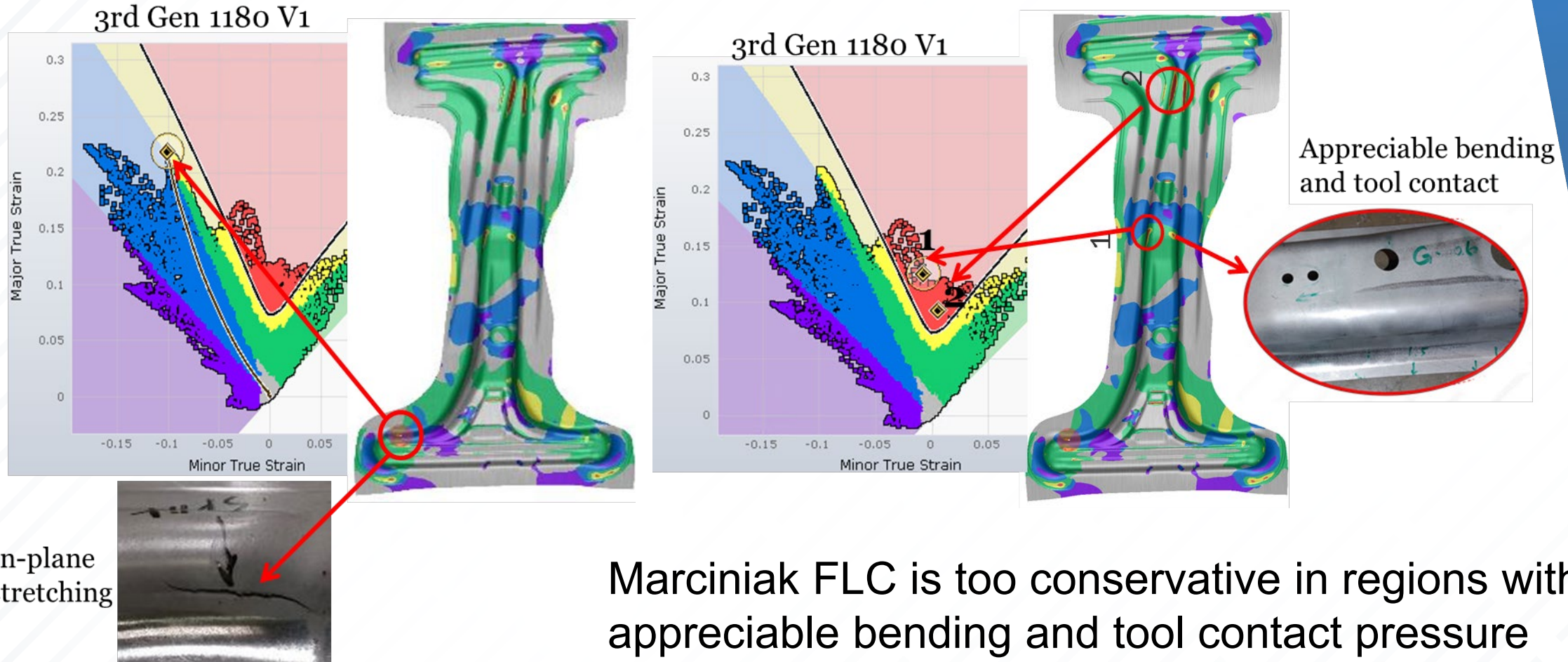
TWENTY YEARS

**NOVEL INSTABILITY FRAMEWORK TO
PREDICT LOCAL FORMABILITY IN 3RD
GEN AHSS B-PILLAR TECHNOLOGY
DEMONSTRATOR**

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University of Waterloo

MOTIVATION

GDIS 2021: Multiple false splits predicted in plane strain tension



OBJECTIVES

Collaborative research project between HDMA, AISI Automotive Program, Bowman Precision Tooling & University of Waterloo

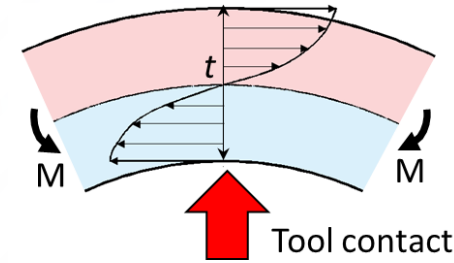
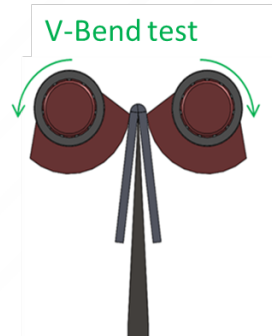
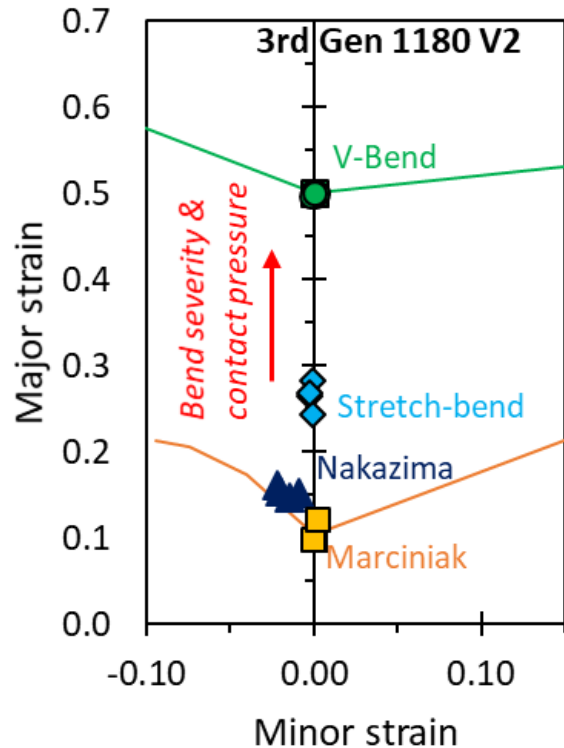
- ❖ Overall goal: Develop CAE predictive capabilities for formability and crash performance of 3rd Gen AHSS and application to mid-size SUV B-Pillar design*

** Separate GDIS 2022 Presentation: “Characterization of 3rd Gen AHSS Towards Reliable Forming and Springback”, Kannan Kidambi*

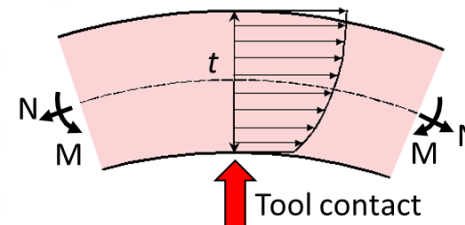
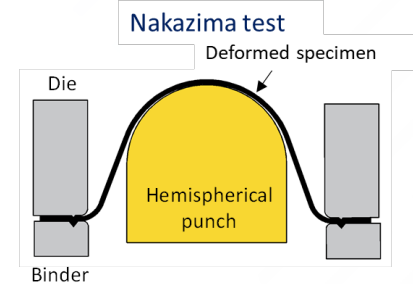
- ❖ State-of-the art Forming Limit Curve (FLC) works well for in-plane stretching
Limitations for assessment of local part feasibility

Objective: Development of instability framework which accounts for instantaneous forming limits in the presence of bending and tool contact pressure

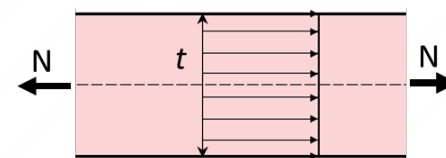
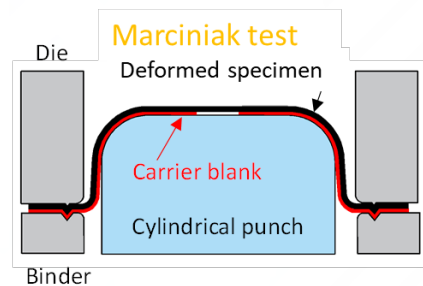
EFFECT OF BOUNDARY CONDITIONS ON NECKING



Severe bending
 Inner layer: compression
Necking suppressed



Combined loading
 Inner layer: strain accumulation delayed
Necking delayed



In-plane stretching
 Homogeneous deformation
Necking occurs simultaneously

Can exploit the delay in plastic instability in the product design stage
 ... Need to develop a framework which accounts for delayed instability

EFFECT OF CONTACT PRESSURE

Hillier [1] instability framework valid for general loading

Physically derived: instability occurs when second order plastic work rate vanishes

Instability : Intersection critical subtangent (z) with material hardening curve

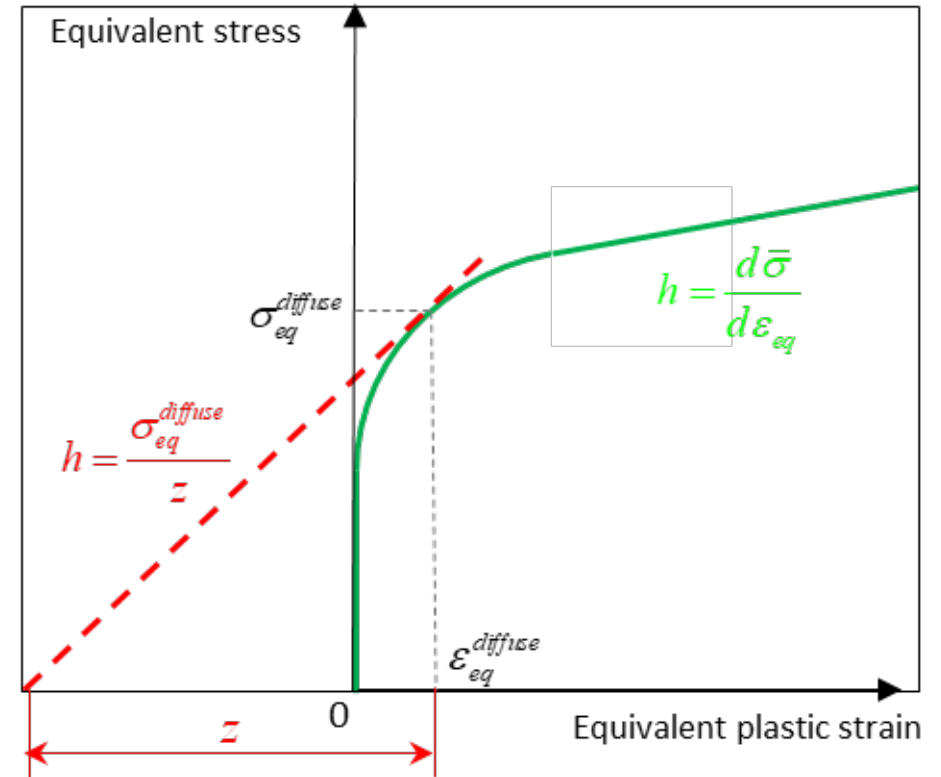
$$\underbrace{\frac{1}{\bar{\sigma}} \frac{d\bar{\sigma}}{d\varepsilon_{eq}^p}}_{\text{Material response}} = \frac{1}{z} = \underbrace{\frac{N_{\delta j}}{\sigma_{eq}} \left(\frac{1}{A_{\delta}} \frac{\partial F_{\delta j}}{\partial \varepsilon_{ml}} \frac{d\varepsilon_{ml}}{d\varepsilon_{eq}^p} + \sigma_{\delta j} \delta_{\delta\beta} N_{\delta\beta} \right)}_{\text{Applied tractions to the material}}$$

$\bar{\sigma}$ Equivalent stress
 ε_{eq} Equivalent strain
 $N_{\delta j}$ Normal vector
 A_{δ} Contact area
 $F_{\delta j}$ Applied load

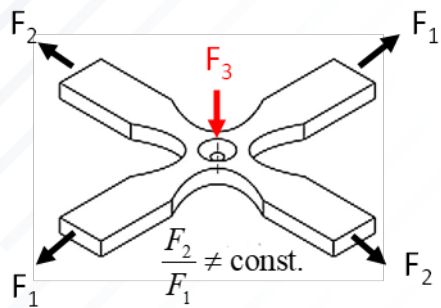
Critical subtangent depends on boundary conditions

Popular 2D instability models (Considère [2], Swift [3]) are special cases of the Hillier [1] framework)

Hillier framework limited to diffuse necking
 ... but can provide insight into how tool contact affects instability



HILLIER GENERAL FRAMEWORK – 3D



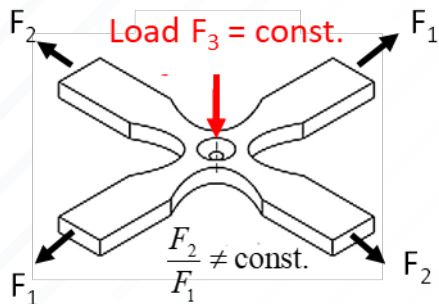
Proportional stressing (P. Stress)

$$\chi = \frac{\sigma_3}{\sigma_1} = \text{const.}$$

$$\frac{1}{z} = k(N_1)^2 (1 + \alpha\rho^2 + \chi\omega^2)$$

Proportional **stressing**: applied forces are adjusted to control for geometric change

Proportional **loading**: forces are applied proportionally; stresses are not proportional

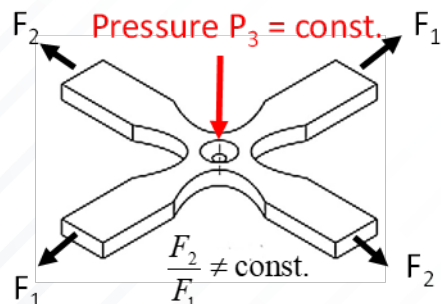
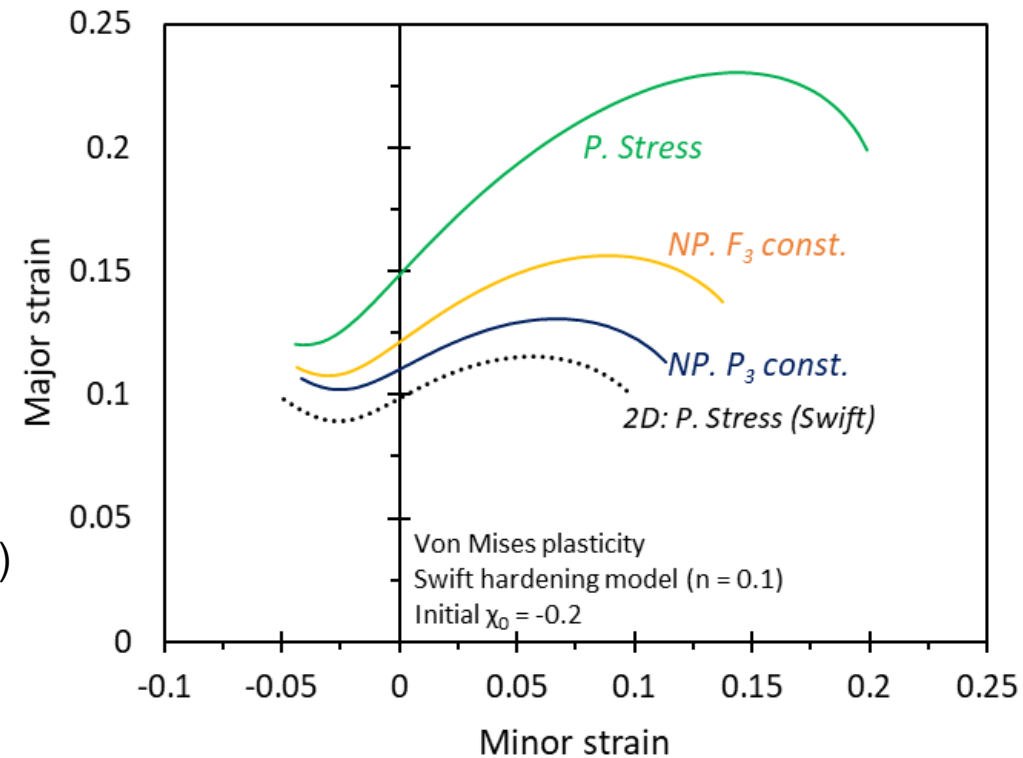


Non-proportional stressing:

Constant normal load (NP. F_3 const.)

$$\chi = \frac{\sigma_3}{\sigma_1} \neq \text{const.} \rightarrow \text{maintain } dF_3 = 0$$

$$\frac{1}{z} = k(N_1)^2 (1 + \alpha\rho^2 + \tilde{\chi}\omega^2)$$



Non-proportional stressing:

Constant normal pressure (NP. P_3 const.)

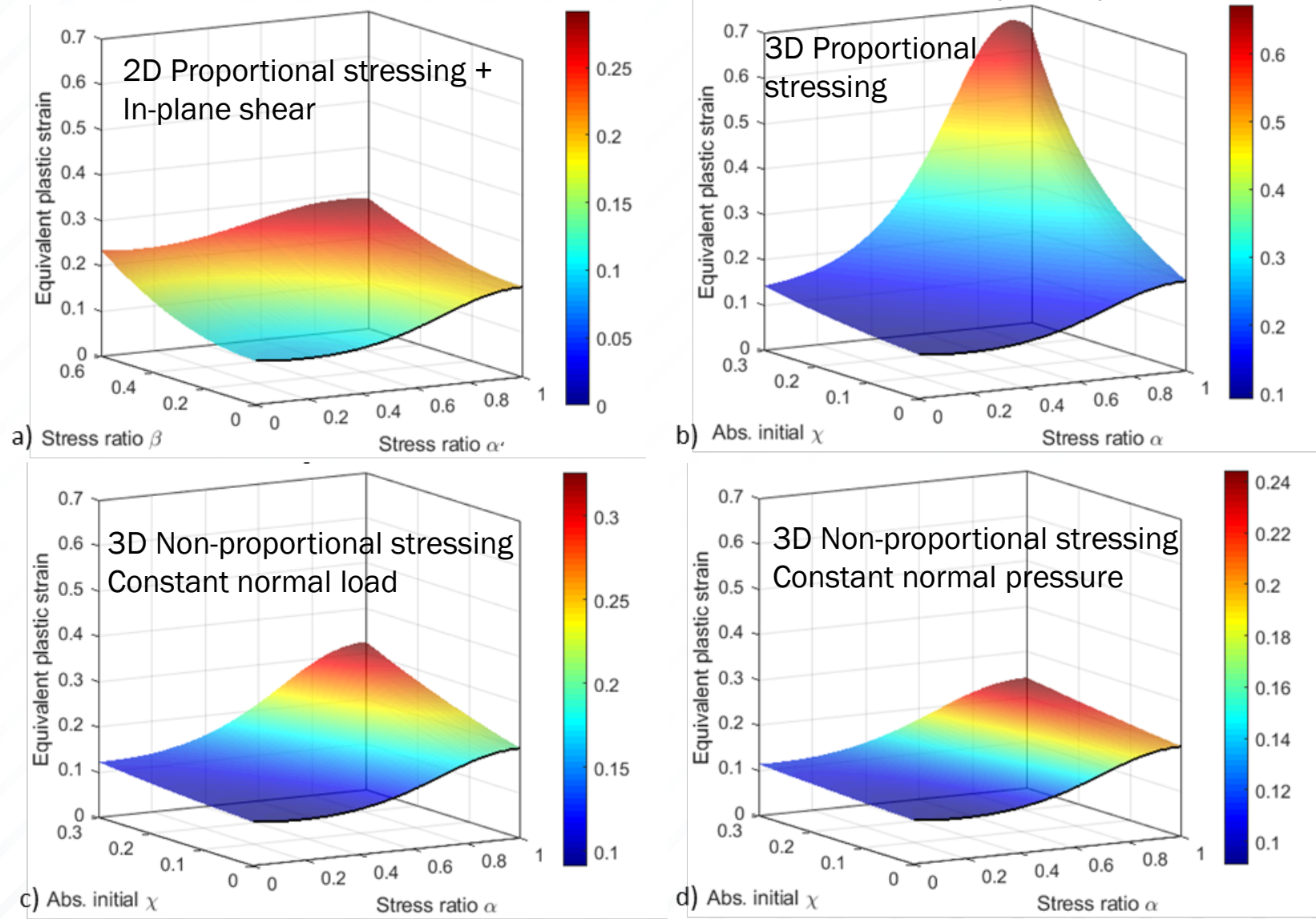
$$\chi = \frac{\sigma_3}{\sigma_1} \neq \text{const.} \rightarrow \text{maintain } dP_3 = 0$$

$$\frac{1}{z} = k(N_1)^2 (1 + \alpha\rho^2)$$

Boundary conditions upon contact pressure govern material instability

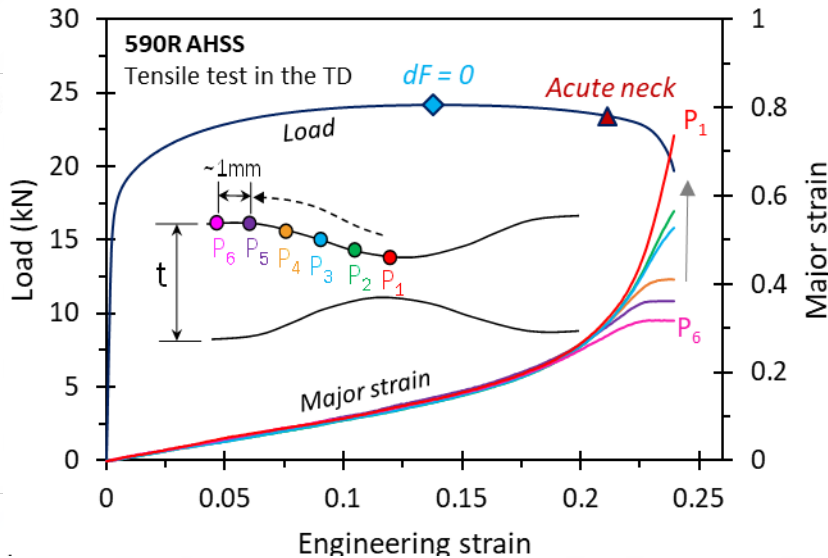
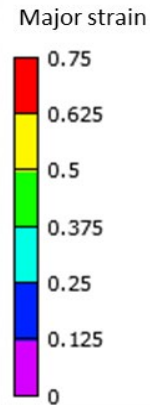
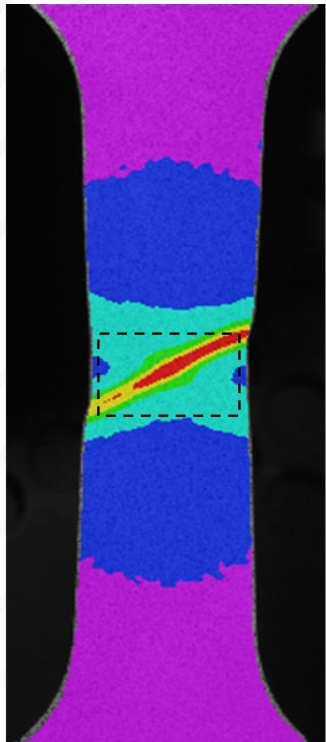
INSTABILITY SURFACES

Plastic instability is an instantaneous metric ... needs to be considered in part feasibility



EXTENSION TO ACUTE LOCALIZATION

Concept of neutral incremental stability



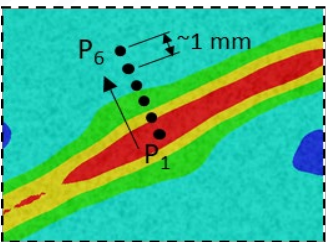
Condition for neutral incremental stability [1]:

$$dW^P \equiv \int_A (dF_i du_i) dA = 0 \quad \text{where } du_i \geq 0$$

du_i Displacement increment
 dF_i Force increment

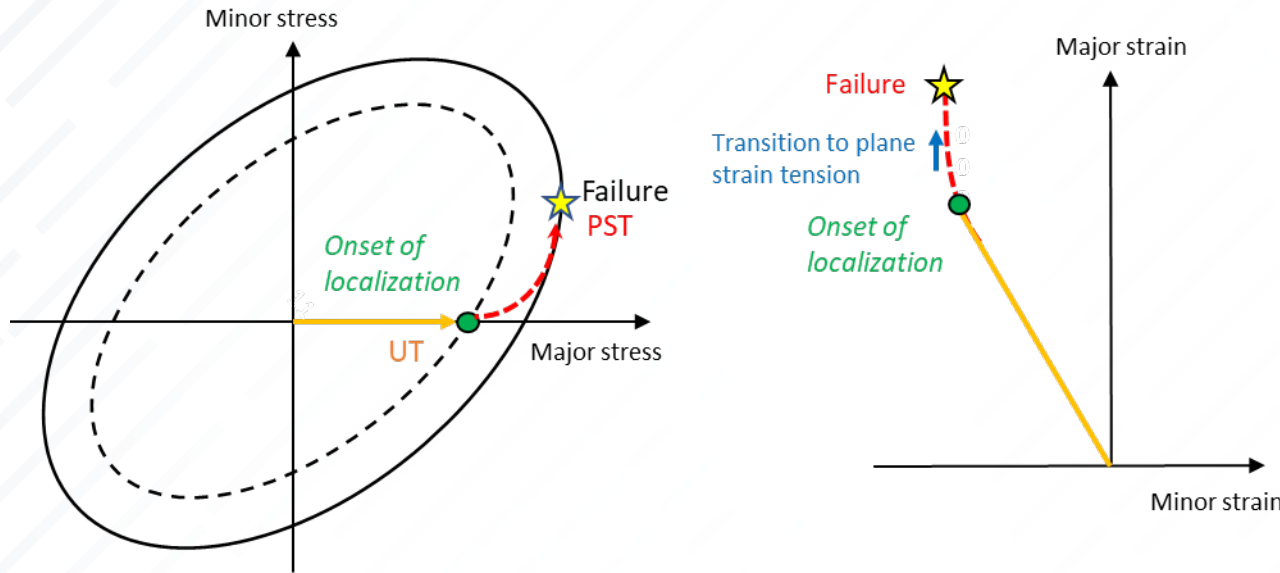
Localization process occurs under neutral incremental stability due to:

- (i) Vanishing load rate or
 - (ii) Vanishing strain rate at the boundary of the band
- or,
- (iii) combination of (i) and (ii)



EXTENSION OF HILLIER MODEL TO ACUTE NECKING

Transition to plane strain tension provides secondary hardening



Assumption for localization:

- (i) Vanishing plastic work rate: stable transition between unstable and stable stress state
- (i) Rate of strain path change is governed by major stress increment to maintain neutral stability

Generalized Incremental Stability Criterion (GISC)

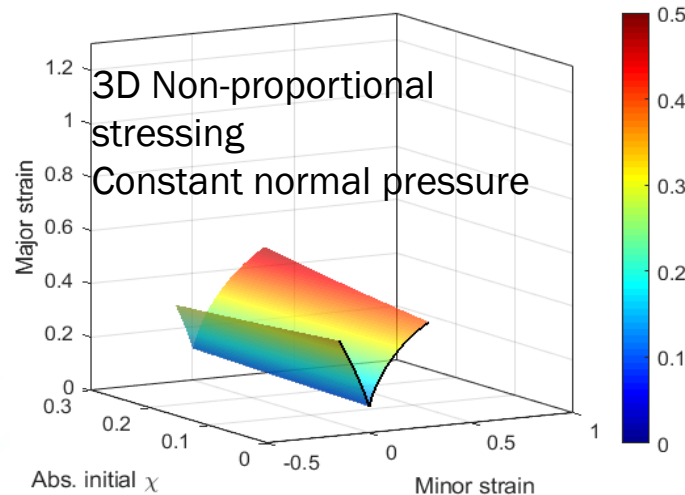
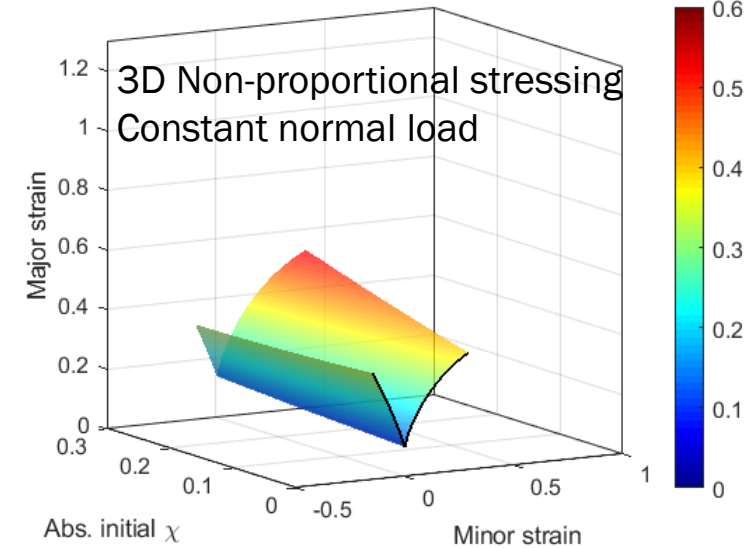
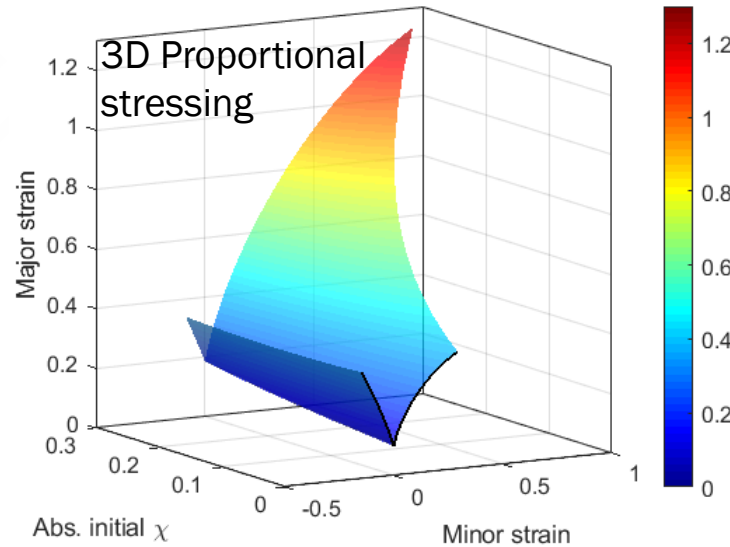
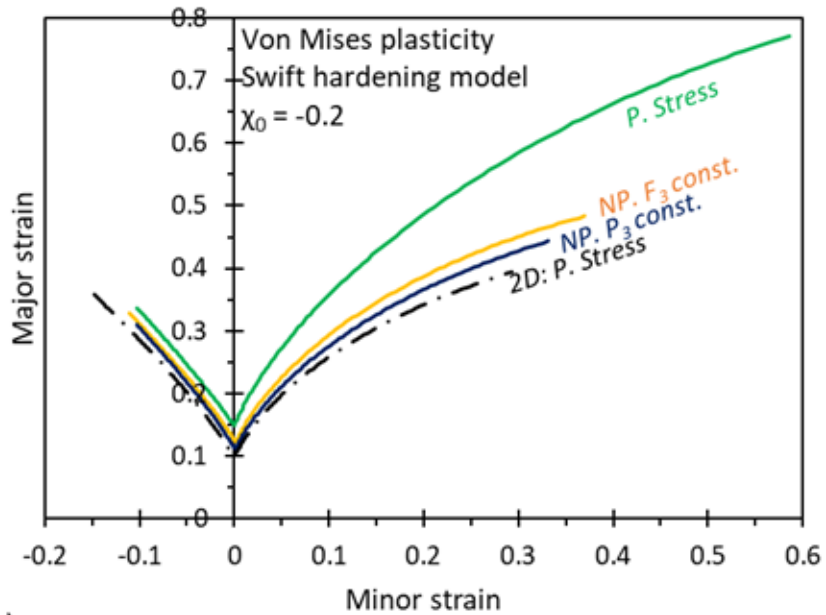
$$\frac{\partial \sigma_1}{\partial \varepsilon_1} + \frac{\partial \sigma_1}{\partial \rho} \geq \frac{1}{z} \left(\frac{k}{N_1} \sigma_{eq} \right)$$

k Ratio of major stress to equivalent stress
 ρ In-plane strain ratio
 N_1 Major normal vector

Valid for principal triaxial loading and accounts for boundary condition of the deformation process

FORMING LIMIT SURFACES

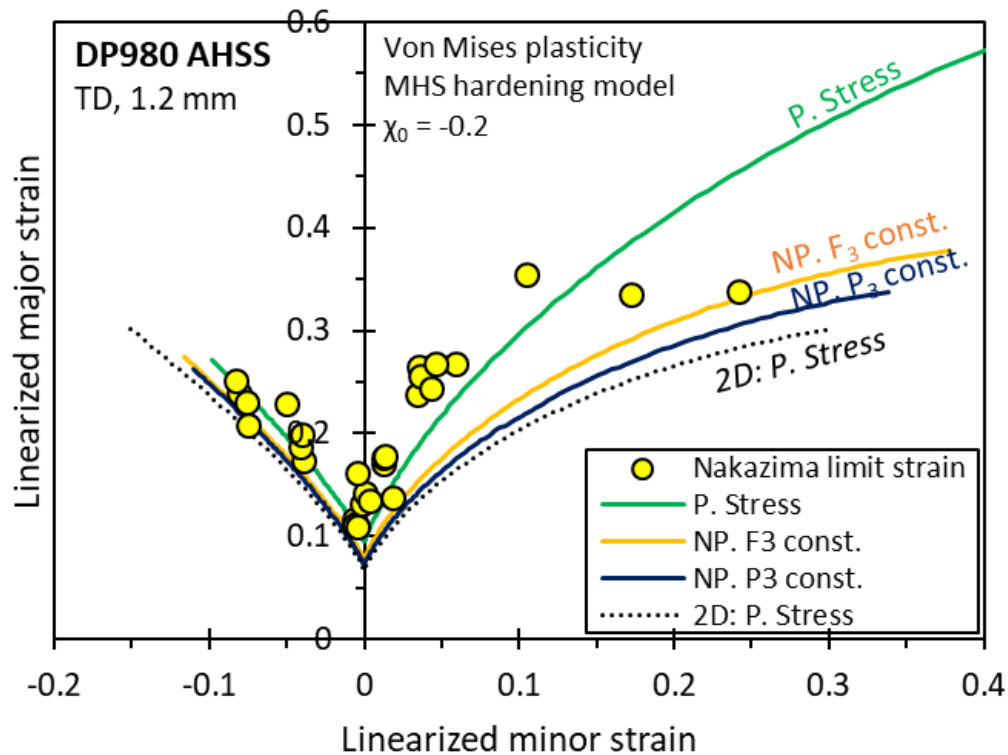
Local boundary conditions significantly affect plastic instability



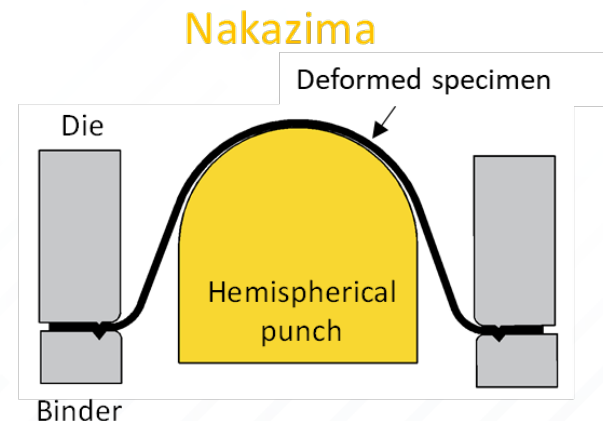
Forming limits should be considered as a surface, specific to the selected boundary condition

IDENTIFICATION OF BOUNDARY CONDITIONS

- ❖ Boundary conditions in formability tests are complex
Material flow is constrained in the plane and sheet is stretched out-of-plane
- ❖ Comparison of studied boundary conditions to formability tests of DP980 AHSS



Assumption of a proportionally evolving contact pressure best captures overall formability trend



EFFECT OF SUPERIMPOSED BENDING

Focus: Plane strain bending + plane strain tensile stretching

Plane strain stretch bending

$$\sigma_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \chi_b \end{bmatrix}$$



Contact pressure

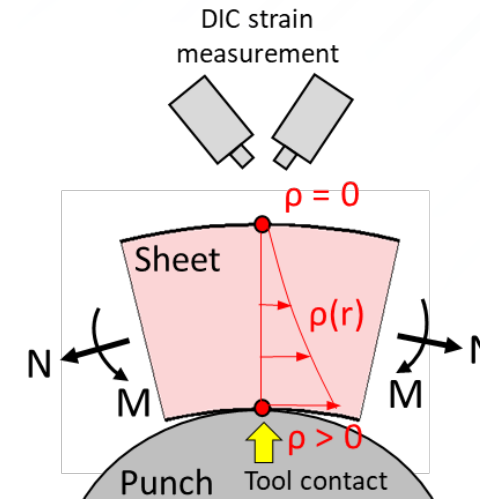
$$\sigma_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_c \end{bmatrix}$$



$$\sigma_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \chi_b + \chi_c \end{bmatrix}$$

Out-of-plane stress ratio: **bending**

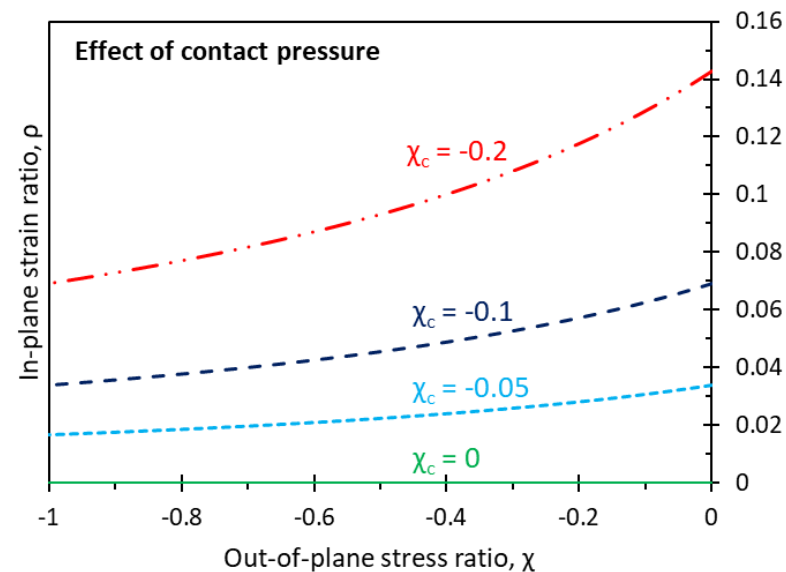
Out-of-plane stress ratio: **contact pressure**



Creates shift of the strain path to positive minor strains (*biaxial shift*)

Introduces a non-linear deformation history for material layers within the cross-section

... challenging for modelling strategy



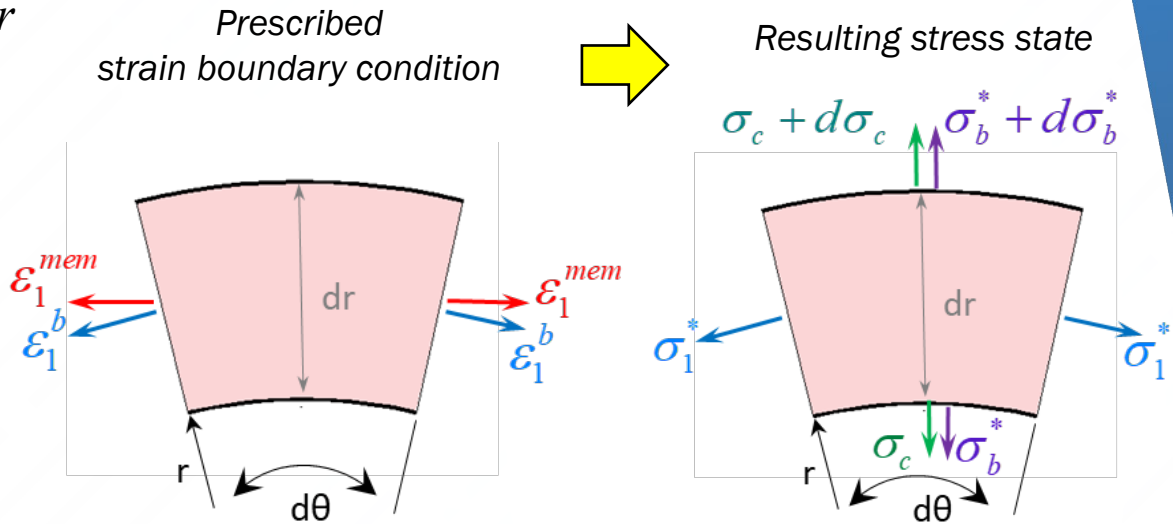
MODEL DEVELOPMENT - 1

Fundamentals of bending mechanics for general loading

$$r \left(d\sigma_b + d\sigma_c - \sigma_b \frac{dw}{w} - \sigma_c \frac{dw}{w} \right) = (\sigma_1 - \sigma_b - \sigma_c) dr$$

Plane strain & no contact pressure

Hill's [3] solution (plane strain bending): $r(d\sigma_b + d\sigma_c) = (\sigma_1 - \sigma_b) dr$



Developed different modelling strategies:

BT Model: accounts for strain path shift to biaxial tension (BT)

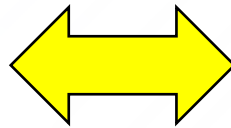
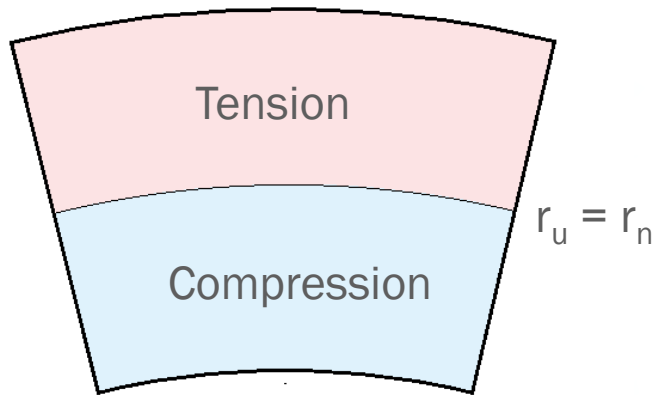
PST Model: enforces plane strain tension (PST)

MODEL DEVELOPMENT - 2

$$r \left(d\sigma_b + d\sigma_c - \sigma_b \frac{dw}{w} - \sigma_c \frac{dw}{w} \right) = (\sigma_1 - \sigma_b - \sigma_c) dr \quad ? \text{ Solving for the ODE}$$

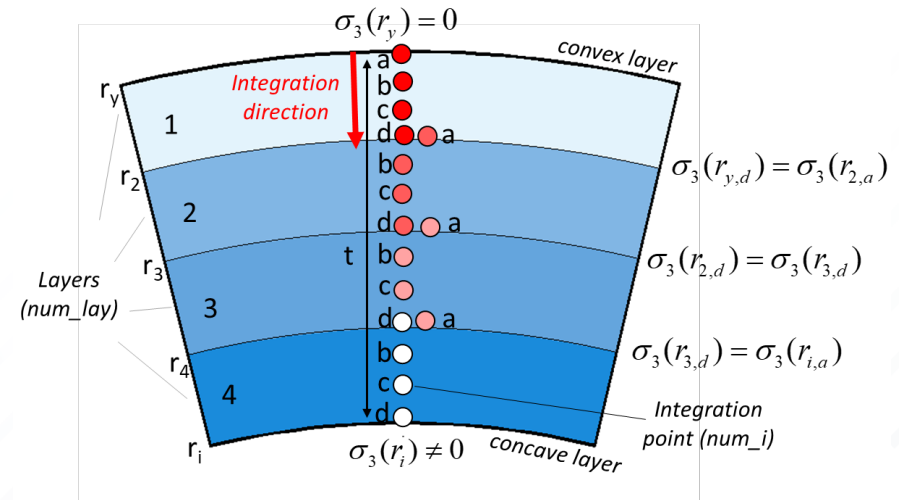
Total stain formulation:
2-zone Model

Closed-form solutions
(von Mises plasticity, Swift hardening)



Incremental stain formulation:
Multi-layer Model

Numerical methods: 4th Order-Runge Kutta



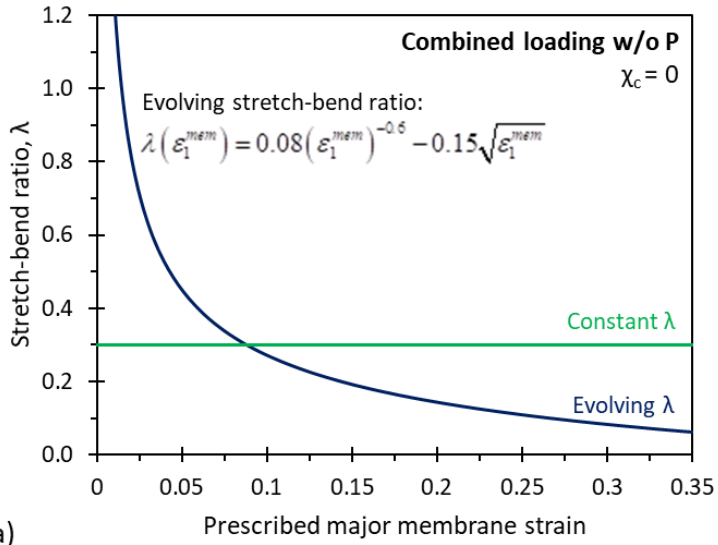
Coupled with the developed instability framework and the concave side rule

COMPARISON – STRAIN FORMULATION

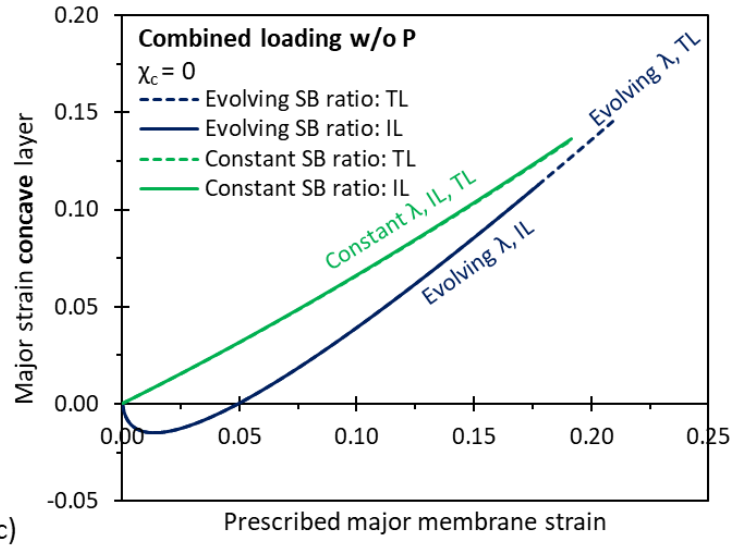


How to prescribe the deformation history?
 Constant stretch-bend ratio (λ) *versus* evolving

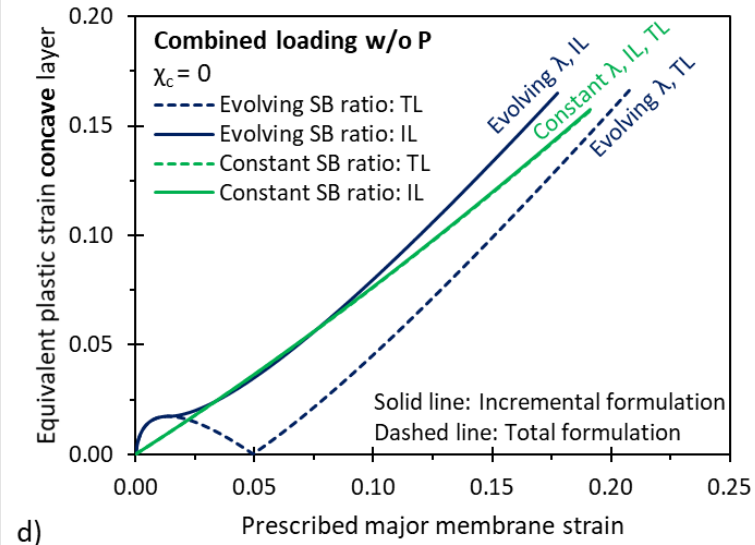
$$\lambda = \frac{d\varepsilon_1^b}{d\varepsilon_1^{mem}} = \frac{\text{Bending}}{\text{Membrane stretching}}$$



a)



c)



d)

- ❖ **Evolving λ** : non-monotonic straining (compression -> tension)
- ❖ Average (**constant**) λ : cross-section undergoes monotonic tensile stretching

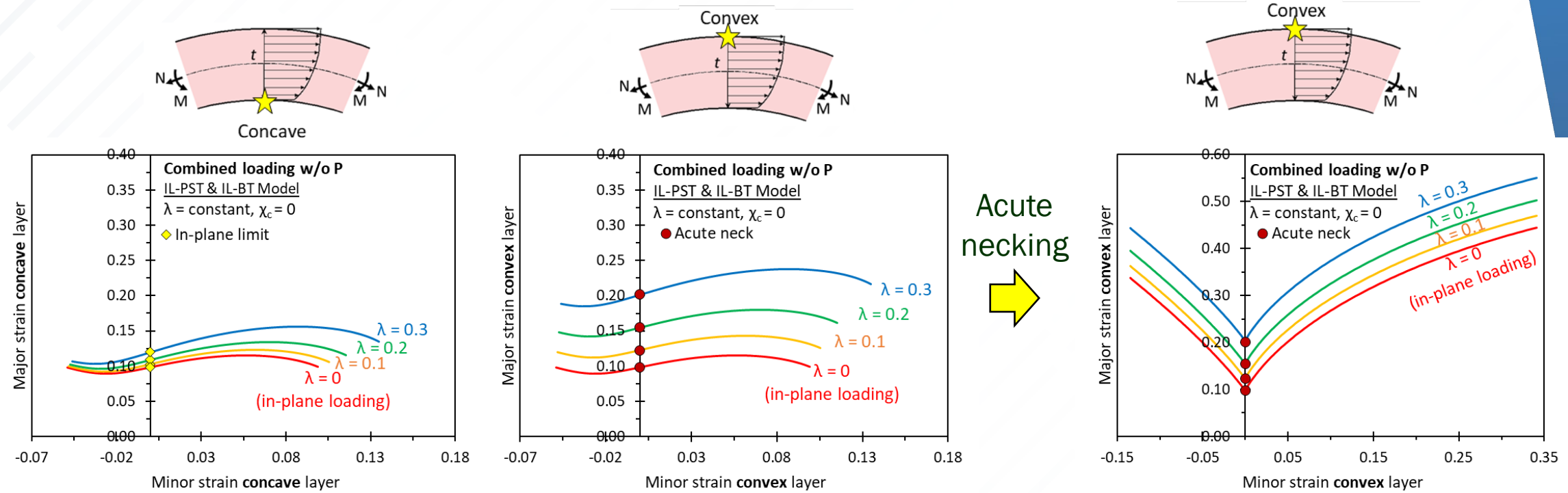
Only incremental strain formulation accounts for cumulative equivalent plastic strain
Total strain formulation erroneously predicts delay in plastic instability

IL: Incremental strain formulation
 TL: Total strain formulation

COMPARISON – MODELLING STRATEGY-1

(1) Combined loading **without** contact pressure (constant λ)

Due to the absence of contact pressure, both models PST and BT are in agreement

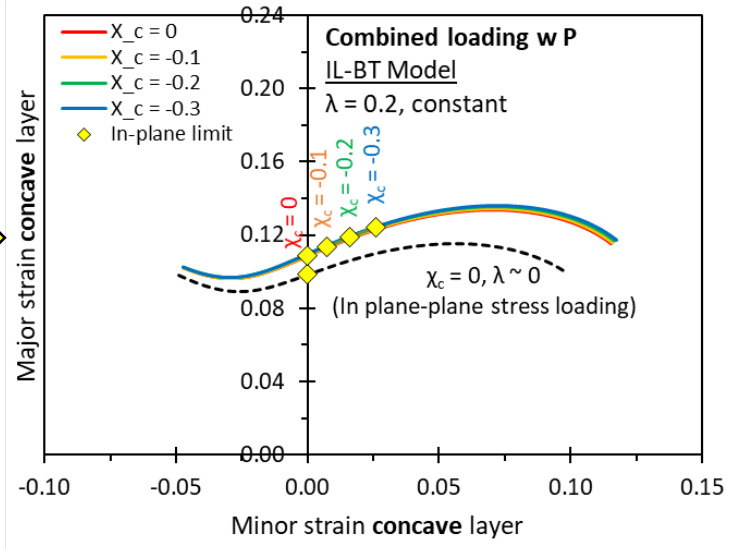
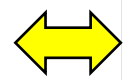
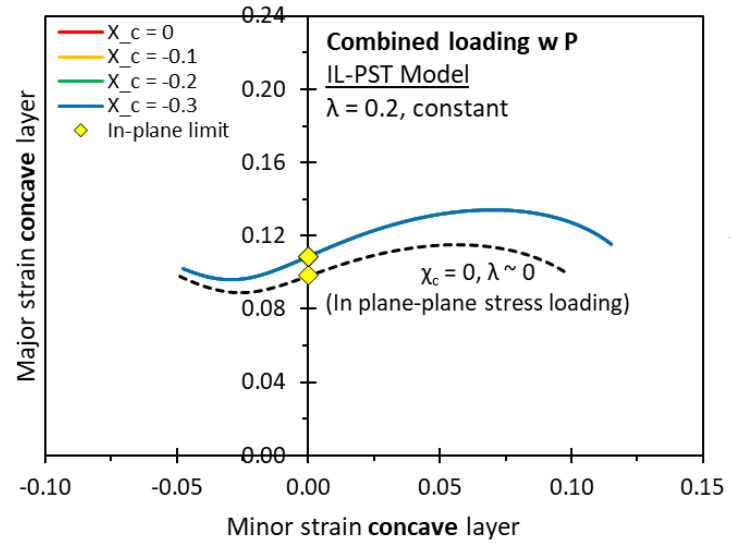
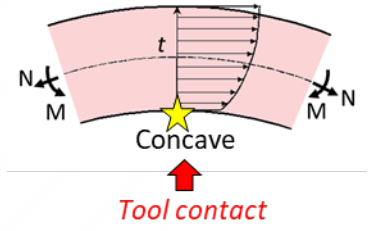
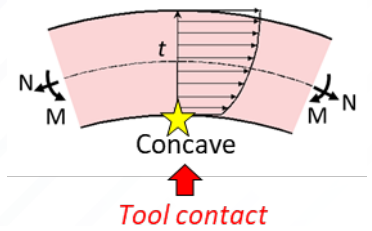


Larger stretch-bend ratio yields higher limit strains since strain accumulation on the concave layer is delayed

BT Model: accounts for strain path shift
 PST Model: enforces plane strain tension

COMPARISON – MODELLING STRATEGY-2

(2) Combined loading **with** contact pressure (constant $\lambda = 0.2$)



How does this affect acute necking limits?

PST Model:

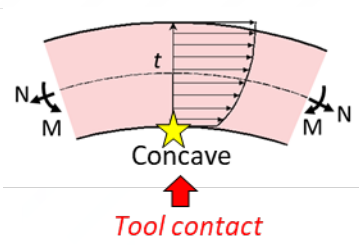
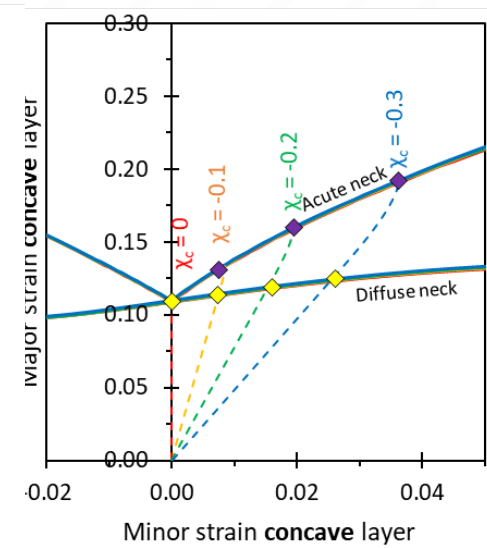
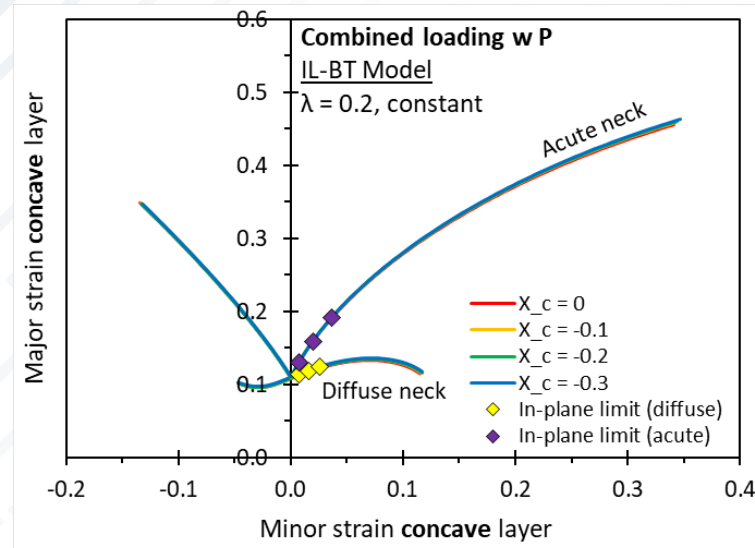
Necking limits are not affected by the presence of contact pressure

BT Model:

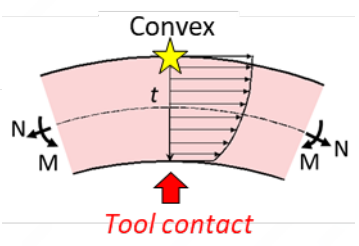
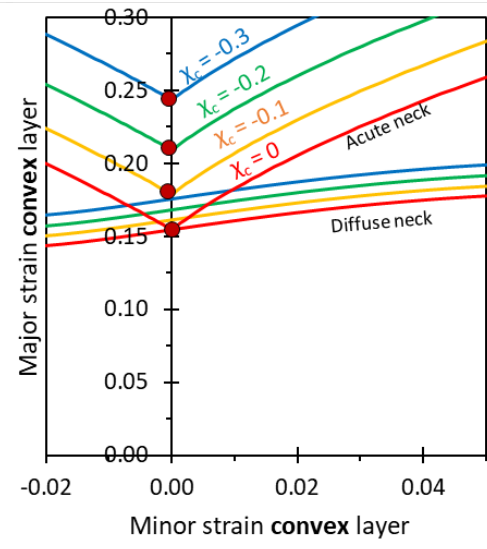
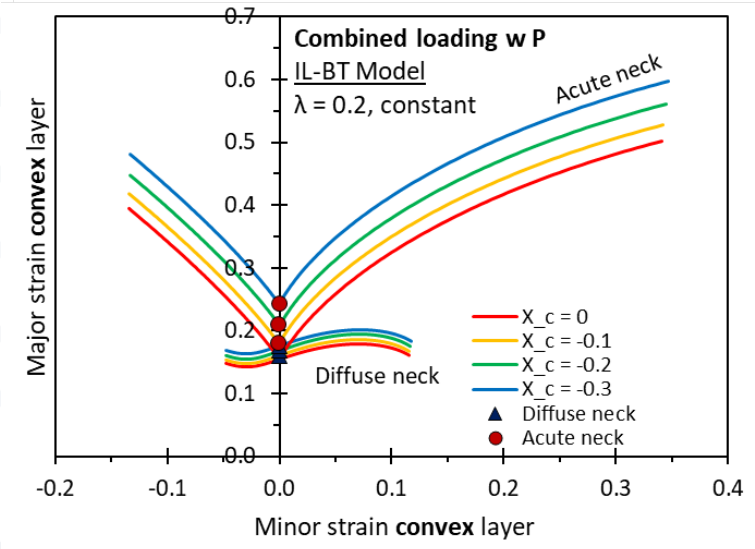
Increase in in-plane limits due to strain path shift caused by contact pressure

BT Model: accounts for strain path shift
PST Model: enforces plane strain tension

COMPARISON – MODELLING STRATEGY-3



In-plane limits on concave layer undergo localization process

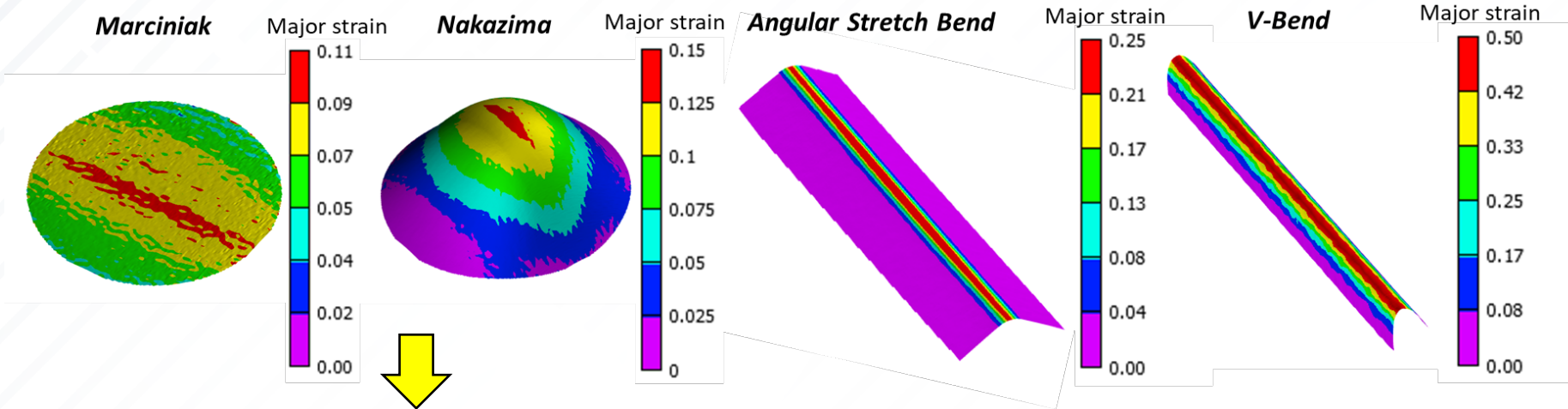


Strain state convex layer \neq concave layer

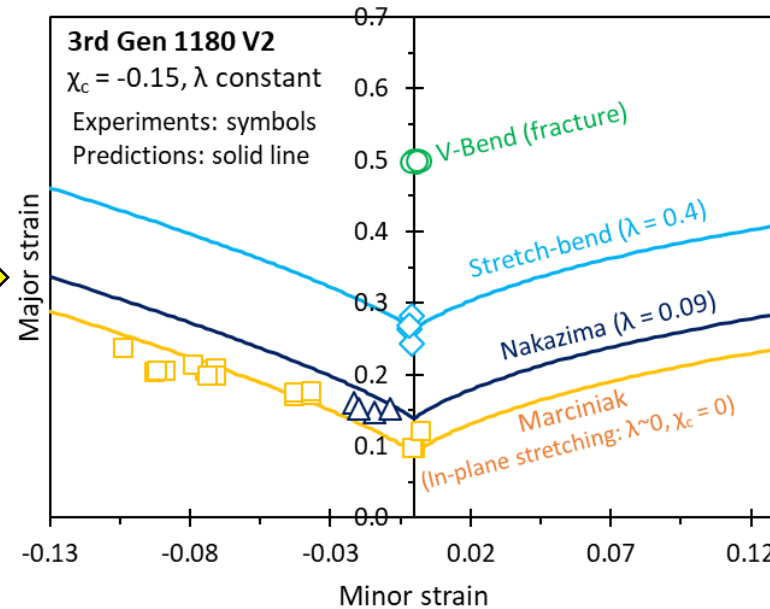
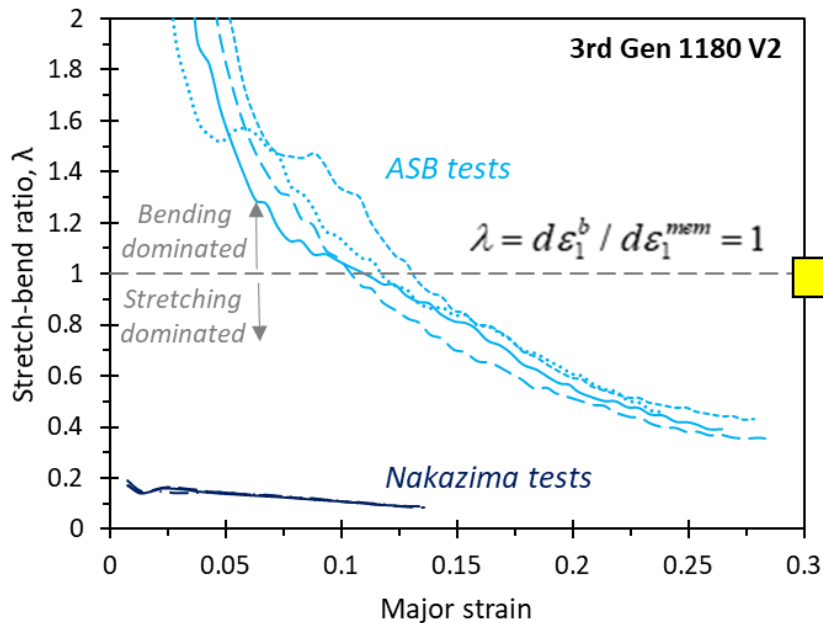
Provides secondary delay in acute necking on the convex layer

* Assume constant formability gain across stress states

APPLICATION TO FORMABILITY TESTS



De-couple measured strain history: Prediction instantaneous forming limits:



Developed instability framework captures overall formability trend well

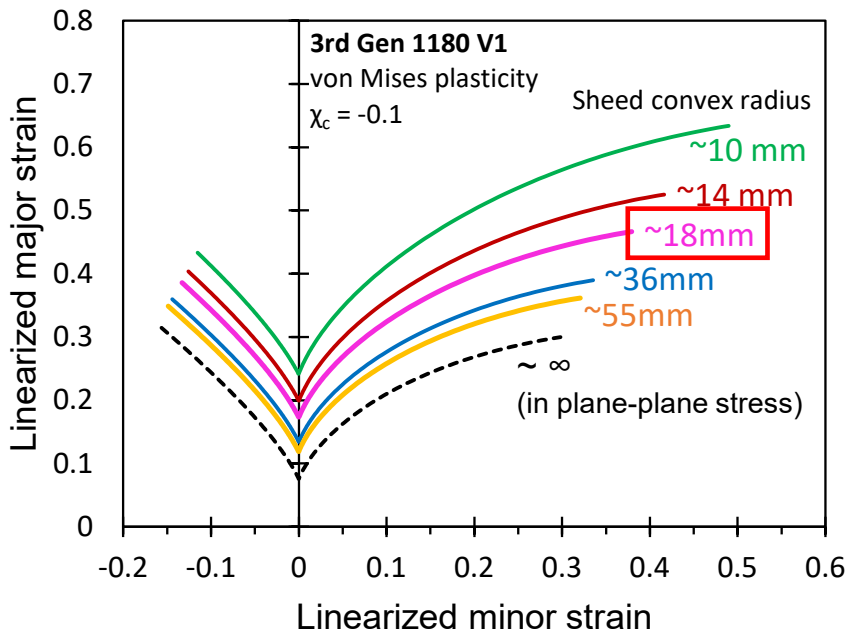
APPLICATION TO TECHNOLOGY DEMONSTRATOR-1



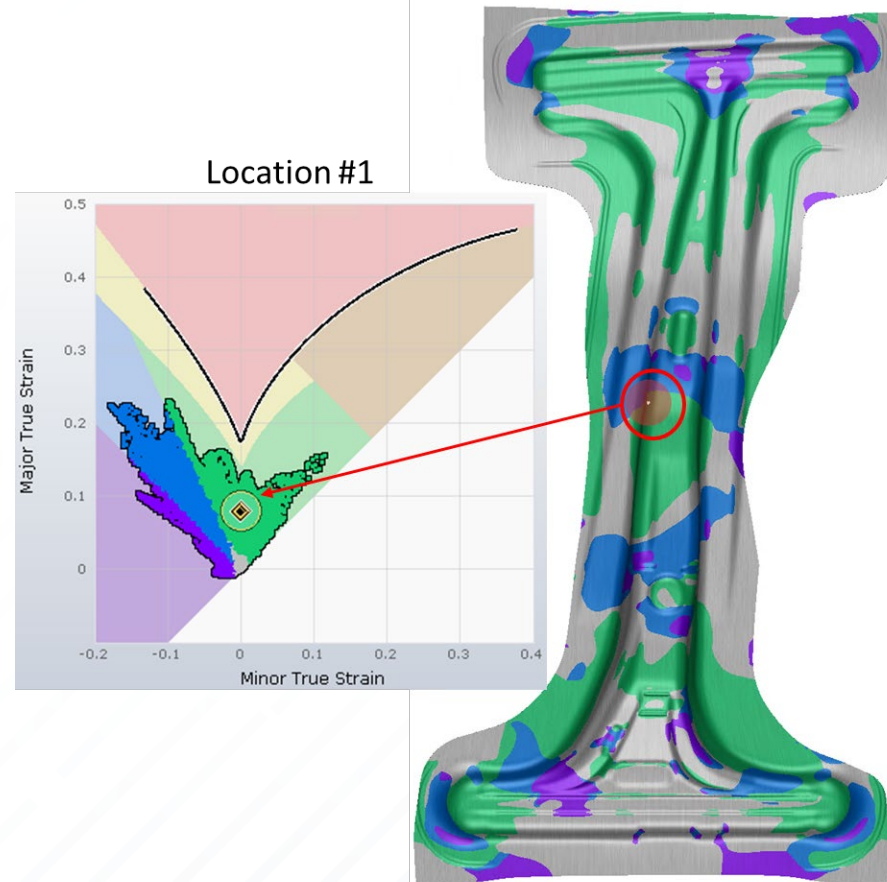
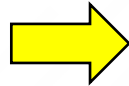
Can now revisit the conservative formability analysis of the 3rd Gen 1180 B-Pillar

Assessment of updated formability analysis

Predict FLC for various bend severities
(Contact pressure obtained in simulation)



Tabulated input of specific FLC in AutoForm software



Bending-enhanced FLC mitigates false positive when using in-plane FLC

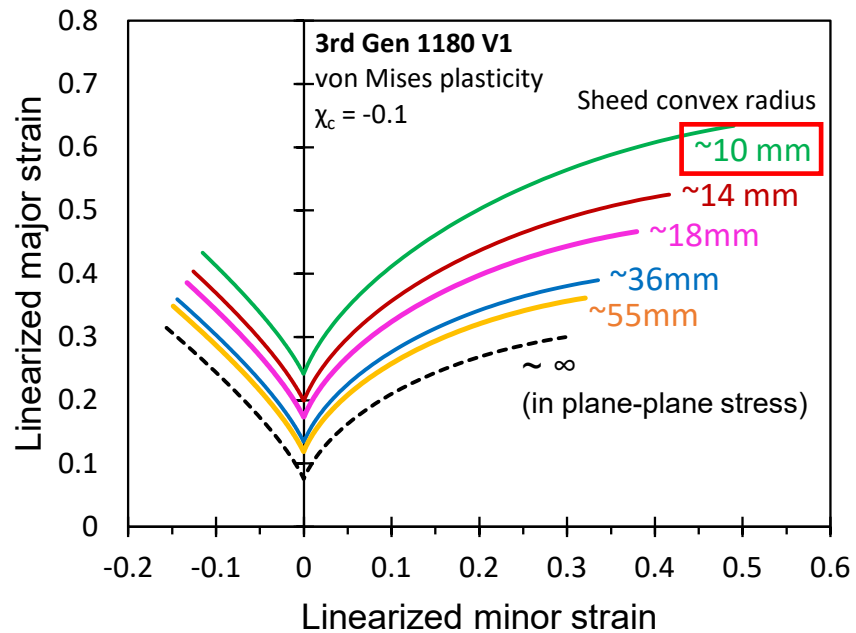
APPLICATION TO TECHNOLOGY DEMONSTRATOR-2



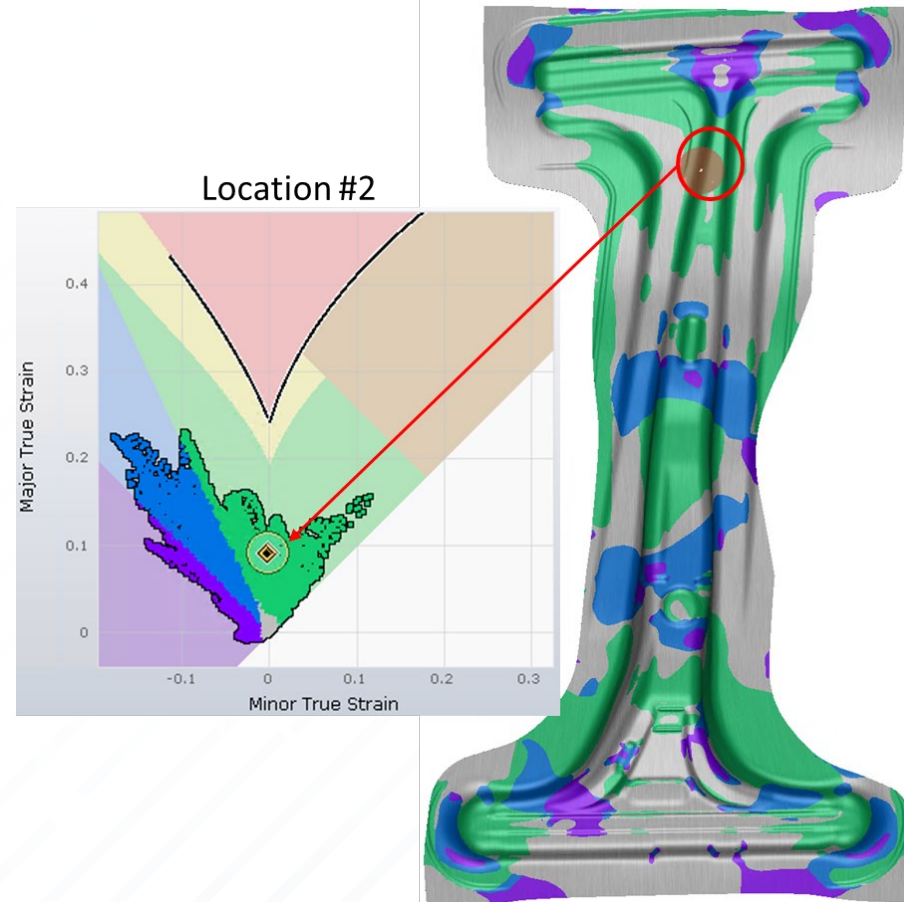
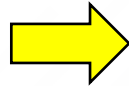
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Bending-enhanced FLC mitigates false positive when using in-plane FLC

CONCLUSIONS

- ❖ Adoption of the conventional in-plane FLC for assessing part feasibility may lead to an overly conservative product design
- ❖ Plastic instability is an instantaneous metric and governed by the boundary conditions of the forming process
Delay in necking (contact pressure, bending effects) can be exploited in the design stage if properly accounted for
- ❖ The developed instability framework (GISC) is physically motivated and can predict acute necking limits under principal triaxial loading considering local boundary conditions
- ❖ Superposition of contact pressure effects to the stress state in plane strain stretch-bending induces a shift of the strain path to positive minor strains which provides a secondary delay in plastic instability
- ❖ Application of the developed modelling strategy to formability tests and a B-Pillar technology demonstrator showed good correlation.



ACKNOWLEDGEMENTS



FOR MORE INFORMATION

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